Math 336, Test 1, Spring 1995

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. The problems are worth 20 points apiece; in the case of a problem with multiple sections, the credit will be divided equally between the parts unless otherwise indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1 State and prove Taylor's theorem.

2 Give examples to show that the following two statements are **FALSE**: a A uniformly continuous function $f :\to$ is bounded. b Every sequence which has a convergent subsequence is bounded above.

3 (a) Carefully state the Weierstrass M-test for the uniform convergence of a series of functions. (b) Show that $\sum_{n=1}^{\infty}(1/n^2)\cos nx$ converges uniformly on to a continuous function.

4 (a) Carefully state the Mean Value Theorem. (b) Suppose that f is a function which is continuous on $[0,\infty)$ and differentiable on $(0,\infty)$. Assume that $f(0) = 0$ and that $f'(x) < 0$ for $x > 0$. Use the Mean Value Theorem to show that $f(x) < 0$ for $x > 0$.

5 Let (f_n) be a sequence of functions on a closed interval $[a, b]$. a (5 pts) Define what it means for (f_n) to converge **pointwise** to a function f. b (5 pts) Define what it means for (f_n) to converge **uniformly** to a function f. c (10 pts) Give an example of a sequence (f_n) which converges pointwise but does not converge uniformly.