Math 336, Test 2, Spring 1995

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. Each problem has the point value indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1 (40 pts) Write an essay at least two pages in length on the theory of integration. State the major definitions in the theory, describe the major theorems, and give the proof of at least one of these theorems.

2 (20 pts) Let (f_n) be a sequence of continuous functions on $[a, b]$, and suppose that $f_n \to f$ uniformly on $[a, b]$. Prove that

$$\lim_{n \to \infty} \int_a^b f_n(x)\, dx = \int_a^b f(x)\, dx.$$

3 (20 pts) Calculate
$$\lim_{n \to \infty} \int_1^2 e^{-nx^2}\, dx.$$

It is probably not a good idea to attempt to evaluate these definite integrals directly. If you make use of any theorem in calculating the limit, give careful and complete arguments to show that the assumptions of the theorem are satisfied.

4 Let f be the following function on $[-1, 1]$: $f(t) = 0$ for $-1 \le t < 0$; $f(t) = 1$ for $0 \le t < 1/2$ or for $1/2 < t \le 1$; $f(1/2) = -1$. Define $F(x)$ for $x \in [-1, 1]$ by
$$F(x) = \int_{-1}^x f(t)\, dt.$$

a (5 pts) Sketch the graphs of $f(x)$ and of $F(x)$. b (5 pts) Where is the function F continuous? c (5 pts) Where is the function F differentiable? Calculate $F'(x)$ for all x at which F is differentiable. d (5 pts) Explain how your answer to (c) fits in with the statement of the second form of the fundamental theorem of calculus.