Math 336, Test 3, Spring 1995

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. Each problem has the point value indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (texbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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"(1)" dom R

1 (20 pts) Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$ have radius of convergence $R > 0$. Prove that f is differentiable on $(-R, R)$ and that

$$f'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1} \qquad for |x| < R.$$

2 (20 pts) Give a careful statement of the Weierstrass Approximation Theorem.

3 (20 pts) Consider the following assertion: *if f is a continuous function on , then there exists a sequence (p_n) of polynomial functions such that (p_n) converges uniformly* **on** *to f*. Show by example that this assertion is **false**. Give careful arguments to show that your example is a relevant one.

4 (a) (10 pts) Give a definition of the log function $L(y)$ in terms of a definite integral. (b) (10 pts) Use this definition to prove that for any two positive numbers y and z, $L(yz) = L(y) + L(z)$.

5 Let $f(x) = x^{\sin x}$ for $x > 0$. (a) (10 pts) Write $f(x)$ in terms of the exponential function E and the log function L. (b) (10 pts) Calculate $f'(x)$.