Math 336, Final Exam, Spring 1996

The exam will be two hours in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. Each problem has the point value indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (textbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

Please write your name at the top of this question sheet and return the sheet inside your exam booklet.

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1 (20 pts) Suppose that f is a function which is continuous on the closed interval $[a, b]$. Prove that f is uniformly continuous on $[a, b]$.

2 (20 pts) Let f be a function which is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). Assume that $f(a) = f(b)$. Prove that there exists a number $c \in (a, b)$ with $f'(c) = 0$. (This is *Rolle's Theorem*).

3 (20 pts) Show that there exists a number x between 0 and 1 such that $2^x = 10x$.

4 (20 pts) Show that if $f'(x)$ exists, then

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h} \ .$$

Bonus (10 extra credit pts): give an example of a function f and a number x such that f is defined in a neighborhood of x and the above limit exists, but $f'(x)$ does **not** exist.

5 (a) (10 pts) State the Taylor Mean Value Theorem. Write out the degree n Taylor expansion of the function $f(x) = e^x$, including the remainder term. (b) (10 pts) Set $x = 1$ in the above Taylor expansion of e^x and show that the remainder term approaches 0 as n approaches infinity. Conclude that $e = \sum_{n=0}^{\infty} 1/n!$.

6 (a) (10 pts) Describe the *integral test* for determining whether or not a series converges. (b) (10 pts) Use the integral test or any other method to determine whether or not the series $\sum_{n=2}^{\infty} 1/n \log n$ converges.

7 (10 pts) Consider the series

$$\sum_{n=1}^{\infty} \left(\frac{(-1)^n}{\sqrt{n}} + \frac{1}{n^2} + \frac{4^n}{(n!)^2} \right) .$$

Determine whether or not the series converges. Give reasons for your answer. (Hint: you may have to break the series up and apply more than one test.)

8(20 pts) Give an example of a bounded function f on the closed interval $[0,1]$ which is **not** integrable. Produce a careful argument to justify the conclusion that $\int_0^1 f$ does not exist. At the very least, you will have to refer to the definition of the integral.