Math 336, Test 1, Spring 1996

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. Each problem has the point value indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (textbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1 (a) (20 pts) State and prove the Intermediate Value Theorem. (b) (10 pts) Does there exist a continuous function whose domain is the closed interval $[0,1]$ and whose range is the entire set $[0,1]\cup[5,6]$? Justify your answer. (c) (10 pts) Does there exist a continuous function whose domain is the closed interval $[0,1]$ and whose range is the entire real line? Justify your answer.

2 (20 pts) (a) (10 pts) State the Mean Value Theorem. (b) (10 pts) Show that for all real numbers a, b there is an inequality

$$|\sin a - \sin b| \leq |a-b| \ .$$

3 (a) (10 pts) Suppose that f is a function defined on a set S of real numbers. Define what it means for f to be *uniformly continuous* on S. (b) (10 pts) Give an example of a function f which is continuous on $(0,1)$ but **not** uniformly continuous on this interval. Justify your answer carefully.

4 (a) (10 pts) Suppose that f is a function on $[a,b]$ with the property that $f^{(n)}$ is continuous on $[a,b]$ and differentiable on (a,b). State the Taylor Mean Value Theorem for this function f. (b) (10 pts) In the situation above, take $a=0$, $b=1/2$, $f(x)=\cos x$ and $n=2$. Write down the expression provided by Taylor's Theorem for $f(b)$.