Math 336, Test 2, Spring 1996

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. Each problem has the point value indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (textbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1 (a) (5 pts) Define what it means for a series to be **absolutely convergent**. (b) (15 pts) Prove that if a series is absolutely convergent, then it converges.

2 (20 pts) Suppose that (a_n) is a sequence of nonnegative numbers, and that there exists a number $\alpha < 1$ such that $(a_n)^{1/n} \leq \alpha$ for all n sufficiently large. Prove that $\sum a_n$ converges.

3 (20 pts) Determine the set of nonnegative numbers x for which the series

$$\sum \frac{x^n}{2^n n!}$$

converges.

4 (20 pts) Suppose that (a_n) is a sequence of nonnegative numbers and that $\sum a_n$ converges. Prove that $\sum (a_n)^2$ converges. (Hint: try comparing one series to the other one.)

5 (20 pts) Give an example of a convergent series $\sum a_n$ such that $\sum (a_n)^2$ does not converge. (By problem 4, it is impossible to find an example like this in which all the numbers a_n are nonnegative. Try constructing $\sum a_n$ as an alternating series.)