Math 336, Test 3, Spring 1996

The test will be 50 minutes in length. Please write your name on the cover of your blue book and write the solutions inside; start the solution to each problem on a new page. Each problem has the point value indicated. This test is being administered under the provisions of the Honor Code. Your work should be your own, and you should not make use of any outside material (textbooks, notes) during the test. What you write should be neat, grammatical, clear and concise. Good luck.

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1 (40 pts) Write an essay at least two pages in length on the theory of integration. State the major definitions in the theory, describe the major theorems, and give the proof of at least one of these theorems.

2 (a) (10 pts) State the Fundamental Theorem of Calculus. (b) (10 pts) Let $f$ be a continuous function on $[-2, \infty)$, and $F$ the function given by $F(x) = \int_{-2}^{x} f(t)\,dt$. Define a function $G$ on $(-\infty, \infty)$ by $G(y) = F(\sin y)$, so that in more explicit terms $G(y) = \int_{-2}^{\sin y} f(t)\,dt$. Use the Fundamental Theorem of Calculus and the Chain Rule to compute $G'(y)$.

3 In the first three parts of this problem, $f$ denotes a continuous function on $[a, b]$ with the property that $f(x) \geq 0$ for all $x \in [a, b]$. We assume in addition that there is a number $c$ in $(a, b)$ such that $f(c) > 0$. Let $y = f(c)$. a (5 pts) Show that there exists a number $\delta > 0$ such that $I = (c - \delta, c + \delta)$ is contained in $(a, b)$ and such that $f(x) > y/2$ for all $x \in I$. b (5 pts) Let $g$ be the function obtained by setting $g(x) = y/2$ for $x \in I$ and $g(x) = 0$ otherwise. Observe that $f(x) \geq g(x)$ for all $x \in [a, b]$. c (5 pts) Conclude that $\int_a^b f \geq (y/2)\delta$; in particular, conclude that $\int_a^b f$ is greater than zero. d (5 pts) Give an example of an interval $[a, b]$ and a function $f$ on $[a, b]$ such that $f$ is integrable, $f(x) \geq 0$ for all $x \in [a, b]$, $f(c) > 0$ for some $c \in [a, b]$, but $\int_a^b f = 0$. Is it possible to find an example like this in which $f$ is continuous?

4 (a) (5 pts) Define the *radius of convergence* of a power series $\sum c_n x^n$.

For each of the following power series, compute both the radius of convergence and the interval of convergence (5 points for each series).

$$(b) \sum_{n=1}^{\infty} x/\sqrt{n} \qquad (c) \sum_{n=0}^{\infty} x^n/3^n \qquad (d) \sum_{n=1}^{\infty} n^2 x^n$$