Homework 10
Math 336, Winter '00
Due Friday, 4/28.

Solve the last three problems on the second exam.

From Wade's book:

- Page 358: 1bd, 6

Other Problems

A. In problem 1b, use the method described in class to approximate a solution (x, y) of $f(x, y) = (.1, 1.2)$. Then use your first approximation to generate a second approximation.

B. This problem is designed to show what can go wrong when the hypotheses of the inverse function theorem fail.

(1) Define $f : \mathbf{R}^2 \to \mathbf{R}^2$ by $f(x, y) = (x^2 + y^2, x - 2y)$.
(2) At which points $(x, y) \in \mathbf{R}$ are the hypotheses of the inverse function theorem satisfied?
(3) Pick any point (your choice) $(x_0, y_0) \in \mathbf{R}^2$ where the hypthoses of the inverse function theorem fail, and let $(r_0, s_0) = f(x_0, y_0)$. Pick a number r slightly larger than r_0. How many real solutions (x, y) of $f(x, y) = (r, s_0)$ are there near (x_0, y_0)? What if r is slightly less than r_0?
(4) Explain why your answers to the last part rule out the existence of an inverse function. Explain what's going on here in geometric terms.

C. The hypotheses of the inverse function theorem can fail even when there exists an inverse function. Consider $f : \mathbf{R} \to \mathbf{R}$. At what points $x \in \mathbf{R}$ do the hypotheses of the inverse function theorem fail? Show that nevertheless, there exists a function $g : \mathbf{R} \to \mathbf{R}$ such that $g \circ f(x) = x$ for every $x \in \mathbf{R}$. What is the difference between this g and the one guaranteed in the conclusion of the inverse function theorem?