Homework 11
Math 336, Winter '00
Due Monday, 5/1.

Solve the last three problems on the second exam.

From Wade's book:

- Page 358: 1bd, 6

Other Problems

A. In problem 1b, use the method described in class to approximate a solution (x, y) of $f(x, y) = (.1, 1.2)$. Then use your first approximation to generate a second approximation.

B. This problem is designed to show what can go wrong when the hypotheses of the inverse function theorem fail.

(1) Define $f : \mathbf{R}^2 \to \mathbf{R}^2$ by $f(x, y) = (x^2 + y^2, x - 2y)$.
(2) At which points $(x, y) \in \mathbf{R}$ are the hypotheses of the inverse function theorem satisfied?
(3) Pick any point (your choice—I want you to use actual numbers here) $(x_0, y_0) \in \mathbf{R}^2$ where the hypthoses of the inverse function theorem fail, and let $(r_0, s_0) = f(x_0, y_0)$. Pick a number r slightly larger than r_0. How many real solutions (x, y) of $f(x, y) = (r, s_0)$ are there near (x_0, y_0)? What if r is slightly less than r_0?
(4) Explain why your answers to the last part rule out the existence of a local inverse for f. Explain what's going on here in geometric terms.

C. Inverse functions can exist even when the hypotheses of the inverse function theorem aren't satisfied. Consider $f : \mathbf{R} \to \mathbf{R}$. At what points $x \in \mathbf{R}$ do the hypotheses of the inverse function theorem fail? Show that nevertheless, there exists a function $g : \mathbf{R} \to \mathbf{R}$ such that $g \circ f(x) = x$ for every $x \in \mathbf{R}$. What is the difference between this g and the one guaranteed in the conclusion of the inverse function theorem?