Homework 6
Math 336, Winter '00
Due Friday, March 3

From the textbook:

- Page 236: 1aecg, 5, 12, 14

Other Problems:

A. This problem is designed to drive home the point that even if a series converges, it doesn't have to do so quickly. Consider the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\log n)^k}$$

(note that the sum begins with $n = 2$ to avoid a zero in the denominator) where k is an integer.

(1) Explain why the series diverges if $k \leq 0$.
(2) Let S_n denote the nth partial sum. Use integrals to give upper and lower bounds for S_n.
(3) Use these bounds to show that the series diverges if $k = 1$ but converges if $k > 1$ (note that S_n is an increasing sequence).
(4) Now suppose that $k = 2$ and denote the sum of the series by S. Use integrals to give upper and lower bounds for $|S_m - S_n|$ for $m > n$. By letting m go to infinity give upper and lower bounds for $|S - S_n|$. How large must n be before these upper and lower bounds differ by less than .01?
(5) If $k = 2$, how many terms of the series would you need to add up to estimate S accurately to within .01?
(6) Compute S accurately to within .01. (*Warning: obviously it'll take awhile to do this if you only try adding up lots of terms. Even your computer would take lifetimes to do it. Take advantage of the estimates you established in part 4 in order to get by with adding up fewer terms.)*