Homework 8
Math 336, Winter '00
Due Friday, March 31

Problem 1: Compute the operator norm of the matrix

$$\begin{pmatrix} 3 & 5 \\ 5 & 3 \end{pmatrix}.$$

(Hint: a vector in $\mathbf{R}^2$ can be written in polar coordinates $(x, y) = (r\cos\theta, r\sin\theta)$).

Problem 2: Show *using the definition of limit* that the function $L : \mathbf{R^2} \to \mathbf{R^3}$ given by

$$L(x, y) = \begin{pmatrix} 1 & -2 \\ e & 1 \\ 1 & \pi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$$

is continuous.

Problem 3: Show that the intersection of finitely many open sets is open. Give an example of a countable collection of open sets $U_j \subset \mathbf{R}^2$, $j \in \mathbf{N}$ whose intersection $\bigcap_{j=1}^{\infty} U_j$ is not open.

Problem 4: Let $f : \mathbf{R}^n \to \mathbf{R}^m$ be a continuous function and let $U \subset \mathbf{R^m}$ be an open set. Show that the set

$$f^{-1}(U) \stackrel{\text{def}}{=} \{\mathbf{x} \in \mathbf{R}^n : f(\mathbf{x}) \in U\}$$

is open.

Problem 5: Let $f, g : \mathbf{R}^n \to \mathbf{R}^m$ be functions and $\mathbf{a} \in \mathbf{R}^n$, be a point at which both $\lim_{\mathbf{x}\to\mathbf{a}} f(\mathbf{x})$ and $\lim_{\mathbf{x}\to\mathbf{a}} g(\mathbf{x})$ exist. Show that $\lim_{\mathbf{x}\to\mathbf{a}} f(\mathbf{x}) + g(\mathbf{x})$ exists and is equal to $\lim_{\mathbf{x}\to\mathbf{a}} f(\mathbf{x}) + \lim_{\mathbf{x}\to\mathbf{a}} g(\mathbf{x})$.

Problem 6: # 2 on page 244 of Wade's book—no proofs necessary.