

Quiz 2

Math 336, Winter '00

1. Let \mathcal{M} be a set and $\rho : \mathcal{M} \times \mathcal{M} \rightarrow \mathbf{R}$ a function. What must ρ satisfy in order to be a metric for \mathcal{M} ? (5 points)

2. Following are four sequences of continuous functions on $[0, 1]$. One sequence converges with respect to the metric $\rho(f, g) = \|f - g\|_\infty$. One of the other two converges in the metric $\rho(f, g) = \int_0^1 |f(x) - g(x)| dx$. Identify these sequences and their limits. (5 points)

(1) $f_n(x) = n(1 - x)^n$

(2) $f_n(x) = \begin{cases} 1 & \text{if } x \geq 1/n \\ nx & \text{if } x < 1/n \end{cases}$

(3) $f_n(x) = \sin(2\pi nx)$

(4) $f_n(x) = \begin{cases} x/n & \text{if } x \leq 1/2 \\ (1 - x)/n & \text{if } x \geq 1/2 \end{cases}$