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1. For any three subsets A, B, C of a universe U prove that

$$(A - B) - C = A - (B \quad C) = (A - C) - (B - C).$$

2. Determine if each of the following mappings is injective, surjective, bijective?

(a) $f\colon \mathbf{Z}^+ \to \mathbf{Z}^+$, $f(n) = \begin{cases} (n+1)/2 & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}$

(b) $g\colon \mathbf{Z} \times \mathbf{Z}^+ \to \mathbf{Q}$, $g(n, m) = \dfrac{n}{m}$

(c) $h\colon \mathbf{R}^+ \to \{x \in \mathbf{R} \mid x > 1\}$, $h(x) = 1 + x^2$.

For all bijective mappings find its inverse.

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3. Factor the following permutation into disjoint cycles.

(a) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 8 & 9 & 5 & 2 & 1 & 6 & 4 & 7 \end{pmatrix}$, (b) $(1\ 2\ 3\ 4\ 5)(6\ 7)(1\ 3\ 5\ 7)(1\ 6\ 3)$

4. Prove that there are no two permutations ρ and σ in S_4 such that simultaneously

$$\rho\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} \text{ and } \sigma\rho = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}.$$

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5. Find the greatest common divisor of the numbers 346 and 461 and represent it as a linear combination of the two numbers

6. Prove using the representation of the greatest common divisor of two integers as their linear combination that if a and b are relatively prime then also a^2 and b are relatively prime.

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7. Prove that if $n \equiv 7 \pmod{12}$, then $n \equiv 3 \pmod{4}$.

8. Find all integers $k \geq 2$, such that $k^2 \equiv 3 \pmod{k}$

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9. Find all solutions of the system of equations

$$\begin{aligned} [3]\,x + [2]\,y &= [1] \\ [5]\,x + \quad y &= [1] \end{aligned} \qquad \text{in } \mathbf{Z}_{11}\,.$$

10. Find all third roots of $8i$. Express them in the standard form.