Student's name:.................................

1. Define:
    (a) Group.

    (b) Abelian group.

    (c) Cyclic group.

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2. Define:

   (a) subgroup

   (b) left coset

   (c) right coset

   (d) Normal subgroup

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3. Define:

   (a) Isomorphism of groups.

   (b) Automorphism

   (c) Inner automorphism

   (d) Homomorphism.

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4. Give a necessary and sufficient condition for a subset H of a group G, to be a subgroup.

How can this condition be weakened, if the subset is finite.

5. A_3 can be treated as a subgroup of A_4 in a natural way, as the set of all even permutations on {1, 2, 3, 4}, which leave 4 unchanged. What is the index of A_3 in A_4? Find the left coset $(1\ 4)A_3$ and the right coset $A_3(1\ 4)$. What permutations are common to the two sets?

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6. Prove: A cyclic group of order n has a subgroup of order m, whenever $m|n$.

7. Let G_1 and G_2 be groups with operations $\cdot$ and $*$ and identity elements e_1 and e_2, respectively. Prove that the set $G_1 \times G_2$ of ordered pairs (x_1, x_2), where $x_1 \in G_1$
and
$x_2 \in G_2$, with operation: $(x_1, x_2)(y_1, y_2) = (x_1 \cdot y_1, x_1 * y_1)$ forms a group.

Prove that every element of the form (x_1, e_2) commutes with every element of the
form
(e_1, y_2).

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8. Let $h: S_n \to \mathbf{Z}_2$ be defines by formula

$$h(\sigma) = \begin{cases} [0] & \text{if } \sigma \text{ is an even permutation} \\ [1] & \text{if } \sigma \text{ is an odd permutation.} \end{cases}$$

Prove that h is a homomorphism and find the kernel.

9. Prove that a kernel of a homomorphism form G to G' is a normal subgroup of G.

10. Prove that if $h: G \to G'$ is a homomorphism, then $h(G)$ is a subgroup of G'.