Student's name:.................................

1. Define:

(a) Factor group G/N.

(b) Direct product $G \infty H$.

(c) Ring

(d) Integral domain

(e) Field

Student's name:..................................

2. State the following theorems:

   (a) The correspondence theorem.

   (b) The first homomorphism theorem

   (c) Cauchy's theorem

Student's name:.................................

3. (a) State the fundamental theorem on finite Abelian groups

(b) Find the number of non-isomorphic Abelian groups of order 108.

4. (a) Define the center $Z(G)$ of a group G.

(b) Find the center of the group D_4 , the dihedral group of order 8.

(c) Find the center of the group M of all 2∞2 non-singular matrices with real entries.

Student's name:..................................

5. (a) Define the centralizer $C_G(a)$ of an element a of G.

(b) Prove that $C_G(a) < G$.

(c) Find the centralizer of the element $a = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ in the group M of 4(c).

6. (a) Define the normalizer $N_G(H)$ of a subgroup H in G.

(b) Find the normalizer $N_G(H)$ of the subgroup $H = \{e, (1\ 2)\}$ in $G = S_4$

Student's name:.................................

7. (a) State Sylow's theorems.

(b) Prove that a group of order 245 must have a normal subgroup.

8. Let $N \triangleleft G$. Prove that if G/N has a normal subgroup $\bar{M}$, then there exist a subgroup M of G, such that $N \triangleleft M \triangleleft G$.

Student's name:.................................

9. Which of the following subsets of the ring M of all $n\infty n$ matrices with real entries forms a subring:

(a) The set of all nonsingular matrices.

(b) The set of all singular matrices.

(c) The set of all upper triangular matrices

(d) The set of all symmetric matrices

(e) The set of all skew-symmetric matrices

10. Show that the set of all matrices of form $\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}$ form a subring S of the ring M of all $2\infty 2$ matrices. Find the zero divisors. Does S have a unity.