1. Decide about each of the following binary relations on **Z** if it is an equivalence relation. If so, describe the equivalence classes. Justify your answer

(a) $x \sim y$ if and only if $y \mid x$,

(b) $x \sim y$ if and only if $x^2 = y^2$,

(c) $x \sim y$ if and only if $x^3 = y^3$,

(d) $x \sim y$ if and only if $x = 2y$,

2. Find the greatest common divisor of 1680 and 208.

3. Find the general solutions of the congruence $6x = 4 \pmod{14}$.

4. Find all complex numbers z such that $z^3 = 27i$.

5. Find the complex number $\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\, i\right)^{1996}$.

6.
Co
mpute
in $\mathbf{Z}_7$:
Error!
.

7. How many distinct <u>non-trivial proper</u> cyclic subgroups are there in the additive group $\mathbf{Z}_{12}$?

8. Which of the following mappings is an epimorphism of additive groups $\mathbf{Z} \varnothing \mathbf{Z}_4$? [Recall that an epimorphism is a surjective homomorphism.]

(a) $n \varnothing [2n]$,

(b) $n \varnothing [n + 1]$,

(c) $n \varnothing [5n]$,

(d) $n \varnothing [n^2]$,

9. $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 2 & 6 \end{pmatrix}$, $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 4 & 1 & 6 & 2 \end{pmatrix}$. Factor $\sigma\tau\sigma^{-1}$ into disjoint cycles.

10. What is the order of the element (1 2 3 4)(5 6 7) in S_7 ?

11. Let H be the subgroup of S_4 generated by the cycle (1 2 3). Find the left coset of H, which contains the element (1 2 3 4).

12. A group G has a subgroup of order 45 and a subgroup of order 75. Find the order of G knowing that $o(G) < 400$.

13. Decide about each of the following rings whether it is a domain, an integral domain, a division algebra or a ring.

(a) $\mathbf{Z}_{11}$

(b) $M_2(\mathbf{Z})$ = the set of 2∞2 matrices with entries from **Z**,

(c) The set matrices of the form $\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$, where a □ **Z**.

(d) $\{2m + 2ni \mid m,n \in \mathbf{Z}\}$ as a subring of **C**,

14. Find the greatest common divisor of the polynomials $x^4 + x^3 + x + 1$ and $x^2 + 1$ over the field of all rational numbers **Q**.

15. Which of the following polynomials in **Q**[x] belongs to the principal ideal generated by $x - 1$?

(a) $x^3 + x^2 + x + 1$,

(b) $x^3 - x^2 + x - 1$,

(c) $x^3 + x^2 + 1$,

(d) $x^2 + 1$

16. The same question in $\mathbf{Z}_3\,[x]$.

17. Over which of the fields below is the polynomial $x^2 - 2$ reducible? Justify your answer.

(a) $\mathbf{Z}_3$,

(b) $\mathbf{Z}_5$,

(c) **Q**,

(d) **R**.