Math 361 Syllabus
Fall, 1997
Instructor; M. Dyer

Textbook Abstract algebra by I. N. Herstein, 3rd edn, (1996) Prentice Hall, Upper Saddle River, New Jersey 07458. ISBN 0-13-374562-7

Syllabus

1. Things familiar and less familiar
1.1 Preliminary Remarks 1.2 Set Theory (set operations, etc) 1.3 Mappings (important functions such as the identity, etc, injective and surjective functions and their properties) 1.4 The set of 1-1 mappings of S onto itself 1.5 The integers (divisibility, division algorithm, greatest common divisors, unique factorization into primes, Euclidean algorithm etc).

I also covered the part of 2.4 on equivalence relations at this time, and left the following two sections as review reading for the students: 1.6 Mathematical induction 1.7 Complex numbers.

2. Groups
2.1 Definitions and examples of groups (definition, abelian groups, additive groups of familiar rings, symmetric groups (including notation in 3.1), dihedral groups, general and special linear groups over familiar fields) 2.2 Some simple remarks (uniqueness of identity element, inverses, notation for powers etc) 2.3 Subgroups (definition and characterizations, subgroup generated by a subset, cyclic subgroup) 2.4 Lagrange's theorem (cosets, index and order of a subgroup, Lagrange's theorem, integers mod n and its additive group and unit group, Euler's theorem, Fermat's theorem) 2.5 Homomorphisms and normal subgroups (definition, examples and basic proerties of homomorphisms, isomorphisms, Cayley's theorem, kernel and image, definition and characterizations of normal subgroups) 2.6 Factor groups (Construction) 2.7 Homomorphism theorems (1st, 2nd and 3rd homomorphism theorem and correspondence theorem on subgroups of a quotient) 2.9 Direct products (definition of direct product)

3. The symmetric group
3.2 Cycle decomposition (cycles, order of permutations, generation by two-cycles) 3.3 Even and odd permutations (definition, sign homomorphism, the alternating group)

4. Rings
4.1 Definition and examples (definition of ring, integral domain, field, division ring, familiar examples (integers, integers mod n, familiar fields, quaternions), subrings 4.2 Some simple results (Uniquenes of zero, multiplication by zero etc) 4.3 Ideals, homomorphisms, quotient rings (definition and construction, isomorphism and correspondence theorems) 4.4 Maximal ideals (maximal ideals in a commutative ring and their relation to fields, examples) 4.5 Polynomial rings (Polynomial rings over a field, degree, divisibility, division algorithm, factor/remainder theorem, greatest common divisors, unique factorization into irreducibles, Euclidean algorithm etc, irreducibles generate maximal ideals. I discussed the structure of the quotient ring $F[x]/(p)$ (for F a field, and p an irreducible polynomial over F) in somewhat more detail than the text, emphasising explicit calculations in such fields). 4.6 Polynomials over the rational numbers (determining the rational roots of a polynomial, Gauss' Lemma, Eisenstein's criterion; for most of this material, I gave different proofs than those in the text) 4.7 Field of quotients of an integral domain. 5.1 Characteristic of a field.

Further group theory
6.1 Simplicity of A_n (different proof than the text). Sylow theorems (statement and examples af applications, but no proof). Fundamental theorem on finite abelian groups (statement and examples without proof).