Test 1 for Honors Algebra III, Math 361

October 12, 1998

Instructions: The test is 50 minutes in length.

1. (10 pts) Let z be the complex number $z = 1 + i$. Find $a, b \in \mathbb{R}$ such that $z^{-1} = a + bi$.

(10 pts) Prove by induction that

$$1 + 2 + \cdots + n = \frac{1}{2}n(n+1).$$

2. (10 pts) Find all groups of order 31. Explain why your list is complete.

(10 pts) Find all **abelian** groups of order 8. Explain why your list is complete.

3. (A) (10 pts) Let p be a prime. Show that the nonzero elements of $\mathbb{Z}_p$ form a group under multiplication.

(B) (10 pts) Assume p is a prime, a a positive integer. Prove with details that

$$a^p \equiv a \bmod p.$$

4. (20 pts) Prove in detail that a group of order 9 is abelian.

5. (20 pts) Let

$$H := \operatorname{rowsp}_{\mathbb{Z}} \begin{bmatrix} 8 & -3 \\ 1 & 4 \end{bmatrix}.$$

Compute the invariant factor decomposition of $\begin{bmatrix} 8 & -3 \\ 1 & 4 \end{bmatrix}$. Thereafter describe a finite abelian group which is isomorphic to $\mathbb{Z}^2/H$.