Test 2 for Honors Algebra III, Math 361

Name: November 23, 1998

Instructions: The test is 50 minutes in length.

1. (10 pts) Prove that there is no permutation σ such that $\sigma(1,2,3)\sigma^{-1} = (1,2)$.

(10 pts) Define the alternating group A_n and find its cardinality.

2. (10 pts) Let $f(x) = x^4 + 3x^3 + 6x^2 + 3x + 5 \in \mathbb{Q}(x)$ and $g(x) = x^4 + x^3 + 11x^2 + x + 10 \in \mathbb{Q}(x)$. Find the greatest common divisor of $f(x)$ and $g(x)$.

(10 pts) Show that $x^4 + x^3 + x^2 + x + 1 \in \mathbb{Q}(x)$ is irreducible.

3. (10 pts) Let $\mathbb{Z}_2$ be the field of two elements and let $R = \mathbb{Z}_2[x]$ be the polynomial ring. Prove that $f(x) = x^3 + x^2 + 1$ is an irreducible polynomial.

(10 pts) What can you say about the ideal $(f(x))$. Give details on the structure of the quotient ring $R/(f(x))$ such as: How many elements does it have, what elements have a multiplicative inverse and are there any zero divisors?

4. (20 pts) Let p be a prime. Show that every group of order p^2 is abelian.

5. (20 pts) Let $R = C[0,1]$ be the ring of real valued continuous functions on the interval $[0,1]$. (You can assume that R is a ring). Show that

$$M = \left\{ f \in R \mid f\left(\frac{1}{2}\right) = 0 \right\}$$

is a maximal ideal. What can you say about R/M?