MATHEMATICS 361 Name ____________________

Test 1

September 17, 1999

1 (20 points)

(a) Let $f : S \to T$ be a mapping. Define carefully what it means to say that (i) f is injective

(ii) f is surjective

(b) Suppose $f : S \to T$, $g : T \to U$ are mappings such that gf is bijective. Show that f is injective and g is surjective.

2 (15 points)

Let f be the element of the symmetric group S_5 given by

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 4 & 5 & 3 \end{pmatrix} .$$

(a) If $g \in S_5$ and $fg = gf$, prove that $g(1) = 1$, or $g(1) = 2$.

(b) Find an element g of S_5 such that $fg = gf$, and $g(1) = 2$.

3 (25 points)

(a) Define the term "group."

(b) Let G be the set of all matrices of the form

$$M_{a,b} = \begin{pmatrix} a & b \\ 0 & a^{-1} \end{pmatrix},$$

where a and b are real numbers, with $a \neq 0$. Provide G with the operation of matrix multiplication. Prove that G is a group. (You may use, without proof, the usual properties of matrix algebra.)

(c) How many elements of order 2 does G have?

4 (20 points)

(a) Let H be a subset of a group G. State carefully conditions under which H will be a subgroup of G.

(b) If H and K are subgroups of a group G, such that $H \cup K$ is also a subgroup of G, show that either $H \supset K$ or $K \supset H$.

5 (20 points)

(a) Define the notion of an equivalence relation on a set S, and explain (without proof) how such a relation leads to a partition on S.

(b) Let H be a subgroup of a group G. If $a, b \in G$, define $a \sim b$ to mean that there exist elements h, h' of H such that $ah = h'b$. Prove that this defines an equivalence relation on G, and show that the equivalence class $[e]$ of the identity element e is H.