MATHEMATICS 361 Name ____________________

Test 2

October 13, 1999 3 questions

1 (40 points)

Suppose G, G' are groups, $\theta : G \varnothing G'$ a mapping.

(a) Define what it means for θ to be a homomorphism.

For the rest of this question, suppose θ *is a homomorphism.*

(b) Define the kernel Ker θ of θ , and show that Ker θ is a normal subgroup of G .

[Rest of question 1 on next page]

For the rest of this question, suppose H *is a subgroup of* G *, and* $H' = \theta(H)$ *the image of* H *under* θ *.*

(c) Show that H' is a subgroup of G' .

(d) Show that the inverse image $\theta^{-1}(H') = \{x \in G | \theta(x) \in H'\}$ is equal to HK (i.e., the set $\{hk | h \in H , k \in K\}$).

2. (30 points)

Let G be the set of all ordered pairs (a,b), where a and b are real numbers, with $a \neq 0$. Provide G with a multiplication, given by

$$(a,b)(c,d) = (ac, bc + d) .$$

Then G is a group, with identity element (1,0) . *[You don't have to prove that statement.]*

(a) If $(a,b) \in G$, find $(a,b)^{-1}$.

(b) Show that the set $H = \{(1,b) | b \in \mathbf{R}\}$ of all elements of the form (1,b) in G is a normal subgroup of G , and that the quotient group G/H is isomorphic with the multiplicative group $\mathbf{R}'$ of all nonzero real numbers.

3.(30 points)

Let G be the multiplicative group of all nonzero rational numbers, and let H , K be the subsets of G given by

$$H = \{2^n | n \in \mathbf{Z}\} ,$$

$$K = \left\{ \frac{a}{b} \mid a , b \in \mathbf{Z} , \text{ a and b both odd} \right\} ,$$

where **Z** denotes the set of all integers. Show that

(a) H and K are subgroups of G .

(b) $G = H \infty K$.