MATHEMATICS 361 Name ____________________

Test 3

November 15, 1999 4 questions

1 (20 points)

(a) Express the permutation (1 3 2 4)(4 5)(2 7 3)(4 5 6) as a product of disjoint cycles, and find its order.

(b) Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 4 & 7 & 1 & 2 & 6 & 8 & 3 \end{pmatrix}$ as a product of transpositions (2-cycles).

(c) If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 1 & 5 & 2 \end{pmatrix}$, is σ^{1999} even or odd?

2. (20 points)

Complete the following statements.

(a) A subset S of a ring R is a subring of R if

(b) A subset H of a ring R is a left ideal of R if

(c) A subset I of a ring R is a two-sided ideal of R if

(d) An element a of a commutative ring R is a zero-divisor in R if

(e) A commutative ring R is an integral domain if

3. (32 points)

Fix an integer n. For integers a, b, set

$$A_{a,b} = \begin{pmatrix} a & b \\ nb & a \end{pmatrix} ,$$

and let $R = \{A_{a,b} \mid a, b \in \mathbf{Z}\}$ (where $\mathbf{Z}$ denotes the ring of integers).

(a) Show that R is a commutative subring of the ring $M_2(\mathbf{Z})$ of all 2×2 matrices with integer coefficients.

(b) By computing the product $A_{a,b}A_{a,-b}$ (or otherwise), show that, if $a^2 \neq nb^2$, then $A_{a,b}$ is not a zero-divisor in R.

(c) Show that, if $a^2 = nb^2$, then $A_{a,b}$ is a zero-divisor in R.

4. (28 points)

Suppose J and K are subsets of a ring R, and let I be the set of all elements a of R with the property that $ax \in K$, for all x in J. Show that

(a) If K is a left ideal of R, then I is also a left ideal of R.

(b) If J, K are both left ideals of R, then I is a two-sided ideal of R.