MATHEMATICS 361 Name ___________________

Test 2

October 11, 2000 4 questions

1 (20 points)

(a) Let H be a subgroup of a group G. Define the index of H in G, and give the relation of this to the orders of H and G. (*Proof not needed.*)

(b) A certain finite group G is known to have a subgroup of order 18 and another subgroup of order 60. What is the minimum possible order for G? (*Give a reason for your answer.*)

(c) If G is an infinite group, with a finite subgroup H, what can you say about the index of H in G? (*Proof not needed.*)

2. (30 points)

Let H be a subgroup of a group G, and a a fixed element of G .

(a) Prove that $Ha \subseteq aH$ if, and only if, $a^{-1}ha \in H$, for all h in H.

(b) Define what it means to say that H is a normal subgroup of G.

(c) If H is a normal subgroup of G, explain what the quotient group G/H is (i.e., explain what its elements are and how they are multiplied).

3.(20 points)

Let Z_{20} be the set of the integers mod 20, and U_{20} the multiplicative group contained in Z_{20} , consisting of the congruence classes $[n]$ modulo 20 with $\gcd(n,20) = 1$.

(a) List all the elements of U_{20} .

(b) List all the elements of the cyclic subgroup H of U_{20} generated by the element [7] .

(c) Find all the distinct left cosets of H in U_{20} .

4. (30 points)

Let G be an abelian group with identity element e , and let n be a fixed positive integer.

(a) Show that the map $\theta : G \to G$ given by

$$\theta(x) = x^n$$

is a homomorphism, being careful to point out where the assumption that G is abelian is used.

(b) If

$$G^{(n)} = \{x^n | x \in G\} ,$$

$$G_{(n)} = \{x \in G | x^n = e\} ,$$

Show that $G^{(n)}$ and $G_{(n)}$ are subgroups of G , and that

$$G/G_{(n)} \cong G^{(n)} .$$

(c) If G is the multiplicative group of all nonzero complex numbers, what is the order of $G_{(n)}$? What is $G^{(n)}$? (*Proof not needed.*)