Math 362 Syllabus
Spring, 1997
Instructor; M. Dyer

Textbook Galois theory by Ian Stewart, 2nd edn, Chapman and Hall, 1992.

Syllabus

1 Ring theory (review) 1.1 General properties of rings 1.2 Characteristic of a field 1.3 Fields of fractions 1.4 Polynomials 1.5 Euclidean algorithm

2 Factorization of polynomials (review) 2.1 Irreducibility (covered unique factorization in euclidean domains, somewhat more generally than the book) 2.2 Test for Irreducibility (Gauss Lemma, reduction mod p, Eisenstein's criterion) 2.3 Zeros of polynomials 2.4 Symmetric polynomials (fundamental theorem stated but not proved)

3 Field Extensions 3.1 Field extensions 3.2 Simple extensions 3.3 Constructing simple extensions 3.4 Classifying simple extensions The Euclidean algorithm 3.2 Roots of polynomials 3.3 Polynomials with integer coefficients

4 The degree of an extension 4.1 The tower law (multiplicativity of extension degrees) 4.2 Algebraic Numbers

5 Ruler and Compass 5.1 Algebraic formulation 5.2 Impossibility proofs (duplication of cube, trisection of angle of an equilateral triangle, squaring the circle (last given modulo the proof of transcendence of π in Chapter 6, which was left as reading), stated but did not prove Gauss' theorem on constructability of regular n-gon (Chapter 17 of book)

7 The idea behind Galois theory

8 Normality and separability 8.1 Splitting fields 8.2 Normality 8.3 Separability 8.4 Formal Differentiation

9 Field degrees and group orders 9.1 Linear independence of homomorphisms (and application to proof of Artin's theorem that degree of a field over fixed subfield of a finite group is the group order)

10 Monomorphisms, automorphisms and normal closure 10.1 K-monomorphisms (results on number of monomorphisms of a finite field extension into its normal closure)

11 The Galois correspondence 11.1 The fundamental theorem (proof)

12.1 A specific example (detailed examination of fundamental theorem for splitting field of t^4-2

over the rationals) I left this as reading, and also discussed in class a second example ($t^5 - 2$ over the rationals).

13 Soluble and simple groups (review) Briefly reviewed 13.1 Soluble Groups 13.2 Simple groups 13.3 p-groups (sketching proof of simplicity of alternating group A_n for $n \geq 5$ and leaving details as reading; proved solvability of finite p-groups, which the students had not seen in 361)

14 Solution of equations by radicals To motivate the problem, I described series of substitutions leading to solutions of cubics and quartics over fields of characteristic not two or three (this is discussed using Galois theory in 15.3 of text, but I took a more naive approach) 14.2 Radical extensions I stated the theorem that polynomial equations are solvable by radicals iff their Galois group is, but proved only that (in characteristic zero) solvable equations have solvable groups 14.3 An insoluble quintic ($t^5 - 6t + 3$ over the rationals)

15 The general polynomial 15.2 Showed insolubility by radicals of general polynomial of degree $n \geq 5$ (I did not do 15.1 (Transcendence degrees) and stated but did not prove algebraic independence of the general polynomial's coefficients i.e. of the elementary symmetric functions)

16 Finite fields16.1 Structure of finite fields 16.2 Multiplicative group. (I also examined in detail in class the Galois correspondence for extensions of finite fields (left as an exercise in the text) and obtained the formula for the number of irreducible polynomials of each degree over a finite field)

19 The fundamental theorem of algebra 19.1 Ordered fields and their extensions (proved the fundamental theorem of algebra).