Test 1 for Honors Algebra IV Math 362

March 2, 1998

Instructions: The test is 60 minutes in length.

1.	(15 pts) Assume the points $(0,0)$ and $(1,0)$ are given in the plane.	Use ruler and compass
	to construct the regular square having vertices at $(0,0)$, $(0,\sqrt{2})$,	$(\sqrt{2}, 0)$ and $(\sqrt{2}, \sqrt{2})$.
	Provide details for your constructions.	

Construction steps:

1.

2.

3.

4.

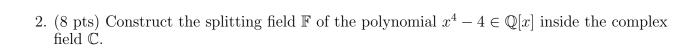
5.

6.

7.

8.

9.



(7 pts) Compute and describe the Galois group $\Gamma[\mathbb{F}:\mathbb{Q}].$

3. (15 pts) Let $\mathbb{F} = \mathbb{Z}_{11}$ be the field of 11 elements. Consider the set of all vectors $x \in \mathbb{F}^{10}$ satisfying

$$\sum_{i=0}^{10} ix_i = 0 \text{ and } \sum_{i=0}^{10} x_i = 0.$$

Compute the cardinality of the set described above.

(5 pts bonus) Show that above 'special ISBN numbers' can correct one error. This means: If at most one of the digits x_i was wrongly recorded as a digit $\tilde{x}_i \neq x_i$ then it is possible to compute the vector $x = (x_1, \dots, x_i, \dots, x_{10})$ from the knowledge of the vector $(x_1, \dots, \tilde{x}_i, \dots, x_{10})$.

4. (15 pts) If $u \in \mathbb{F}$ is algebraic of odd degree over \mathbb{K} , then so is u^2 and $\mathbb{K}(u) = \mathbb{K}(u^2)$.

5. (20 pts) If $f \in \mathbb{K}[x]$ has degree n and f is a splitting field of f over \mathbb{K} then $[\mathbb{F} : \mathbb{K}]$ divides n!.

