**Test 1 for Honors Algebra IV Math 362**

March 2, 1998

Instructions: The test is 60 minutes in length.

1. (15 pts) Assume the points $(0,0)$ and $(1,0)$ are given in the plane. Use ruler and compass to construct the regular square having vertices at $(0,0)$, $(0,\sqrt{2})$, $(\sqrt{2},0)$ and $(\sqrt{2},\sqrt{2})$. Provide details for your constructions.

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Construction steps:

1.

2.

3.

4.

5.

6.

7.

8.

9.

2. (8 pts) Construct the splitting field $\mathbb{F}$ of the polynomial $x^4 - 4 \in \mathbb{Q}[x]$ inside the complex field $\mathbb{C}$.

(7 pts) Compute and describe the Galois group $\Gamma[\mathbb{F} : \mathbb{Q}]$.

3. (15 pts) Let $\mathbb{F} = \mathbb{Z}_{11}$ be the field of 11 elements. Consider the set of all vectors $x \in \mathbb{F}^{10}$ satisfying

$$\sum_{i=0}^{10} i x_i = 0 \text{ and } \sum_{i=0}^{10} x_i = 0.$$

Compute the cardinality of the set described above.

(5 pts bonus) Show that above 'special ISBN numbers' can correct one error. This means: If at most one of the digits $x_i$ was wrongly recorded as a digit $\tilde{x}_i \neq x_i$ then it is possible to compute the vector $x = (x_1, \ldots, x_i, \ldots, x_{10})$ from the knowledge of the vector $(x_1, \ldots, \tilde{x}_i, \ldots, x_{10})$.

4. (15 pts) If $u \in \mathbb{F}$ is algebraic of odd degree over $\mathbb{K}$, then so is $u^2$ and $\mathbb{K}(u) = \mathbb{K}(u^2)$.

5. (20 pts) If $f \in \mathbb{K}[x]$ has degree $n$ and $f$ is a splitting field of $f$ over $\mathbb{K}$ then $[\mathbb{F} : \mathbb{K}]$ divides $n!$.

6. (5 pts) Construct a field $\mathbb{F}$ of 8 elements.

(5 pts) Show that every field of 8 elements must have $\mathbb{Z}_2$ as a prime field.

(5 pts) Determine all intermediate fields of $[\mathbb{F} : \mathbb{Z}_2]$.

(5 pts) Describe a nontrivial field automorphism $\varphi : \mathbb{F} \longrightarrow \mathbb{F}$.

(5 pts bonus) Compute the Galois group $\Gamma[\mathbb{F} : \mathbb{Z}_2]$.