Test 2 for Honors Algebra IV Math 362

April 3, 1998

Instructions: The test is 60 minutes in length.

1. (20 pts) Assume $\mathbb{F}$ is a subfield of the complex numbers, i.e. $\mathbb{F} \subset \mathbb{C}$. Assume $\sigma_1, \sigma_2, \sigma_3$ are three monomorphisms from $\mathbb{F} \to \mathbb{C}$. Assume we have the identity $\sigma_3(x) = 2\sigma_2(x) - \sigma_1(x)$ for all $x \in \mathbb{F}$. What can you say about these three monomorphisms. Explain.

2. (10 pts) Write down an irreducible polynomial which is not separable. Explain why your example has the stated properties.

(10 pts) Write down an irreducible polynomial which is not normal. Explain why your example has the stated properties.

3. (10 pts) Describe in detail the splitting field of the polynomial $x^5 - 1 \in \mathbb{Q}[x]$.

(10 pts) Compute the Galois group and write down the Galois correspondence indicating on one side the subgroups of the Galois group and on the other side the corresponding subfields.

4. (10 pts) Compute the normal closure N of the field extension $\mathbb{Q}\left(\sqrt[3]{2}\right) \supset \mathbb{Q}$ inside the complex numbers $\mathbb{C}$.

(10 pts) Compute the Galois group $\Gamma[N : \mathbb{Q}]$. Hint: You either can describe the Galois group in an explicit way or you can find it by some general arguments.

5. (20 pts) Let $GF(1024)$ be the finite field of 1024 elements. Describe explicitly the automorphisms of the Galois group $\Gamma[GF(1024) : \mathbb{Z}_2]$. Then work out the Galois correspondence indicating on one side the subgroups of Γ and on the other side the subfields of $GF(1024)$.