Math 362 Syllabus
Spring, 1999
Instructor; M. Dyer

Textbook Galois theory by Ian Stewart, 2nd edn, Chapman and Hall, 1992. This text was out of print but I managed to locate a copy for the one student in the class.

Remark I ran the course as a reading course (there was only one student), and spent class time answering questions, giving complements to the reading, doing examples and exercises, and having the student do through exercises. Additional homework exercises were assigned as well. In this way, most of the unstarred exercises (and a few starred ones) in Chapters 1–5 and 7–16 in the text were covered.

Syllabus

1 Ring theory (review) 1.1 General properties of rings 1.2 Characteristic of a field 1.3 Fields of fractions 1.4 Polynomials 1.5 Euclidean algorithm

2 Factorization of polynomials (review) 2.1 Irreducibility 2.2 Test for Irreducibility (Gauss Lemma, reduction mod p, Eisenstein's criterion) 2.3 Zeros of polynomials

3 Field Extensions 3.1 Field extensions 3.2 Simple extensions 3.3 Constructing simple extensions 3.4 Classifying simple extensions The Euclidean algorithm 3.2 Roots of polynomials 3.3 Polynomials with integer coefficients

4 The degree of an extension 4.1 The tower law (multiplicativity of extension degrees) 4.2 Algebraic Numbers

5 Ruler and Compass 5.1 Algebraic formulation 5.2 Impossibility proofs (duplication of cube, trisection of angle of an equilateral triangle, squaring the circle (last given modulo the proof of transcendence of π in Chapter 6), stated but did not prove Gauss' theorem on constructability of regular n-gon (Chapter 17 of book)

7 The idea behind Galois theory

8 Normality and separability 8.1 Splitting fields 8.2 Normality 8.3 Separability 8.4 Formal Differentiation

9 Field degrees and group orders 9.1 Linear independence of homomorphisms (and application to proof of Artin's theorem that degree of a field over fixed subfield of a finite group is the group order)

10 Monomorphisms, automorphisms and normal closure 10.1 K-monomorphisms (results on number of monomorphisms of a finite field extension into its normal closure)

11 The Galois correspondence 11.1 The fundamental theorem (proof)

12.1 A specific example (detailed examination of fundamental theorem for splitting field of t^4-2 over the rationals) Discussed in class in detail a second “large” example (t^5-2 over the rationals).

13 Soluble and simple groups (review) 13.1 Soluble Groups 13.2 Simple groups 13.3 p-groups, simplicity of alternating group A_n for $n \geq 5$)

14 Solution of equations by radicals To motivate the problem, I described series of substitutions leading to solutions of cubics and quartics over fields of characteristic not two or three (this is discussed using Galois theory in 15.3 of text, but I took a more naive approach) 14.2 Radical extensions; proved the theorem that polynomial equations in characteristic zero are solvable by radicals only if their Galois group is solvable 14.3 An insoluble quintic (t^5-6t+3 over the rationals)

15 The general polynomial15.1 Transcendence degree 15.2 The general polynomial Showed insolubility by radicals of general polynomial of degree $n \geq 5$ 15.3 Solving quartic equations (Showed solvable Galois group implies an equation solvable by radicals in characteristic zero; did not discuss explicit solution of cubics and quartics using ideas from Galois theory, after giving the earlier naive treatment of them)

16 Finite fields16.1 Structure of finite fields 16.2 Multiplicative group (also examined in detail the Galois correspondence for extensions of finite fields (left as an exercise in the text) and obtained the formula for the number of irreducible polynomials of each degree over a finite field)

19 The fundamental theorem of algebra 19.1 Ordered fields and their extensions (proved the fundamental theorem of algebra using ideas from Galois theory).