Math 362 Syllabus
Spring, 2001
Instructor: Warren Wong

Textbook Galois theory by Ian Stewart, 2nd edn, Chapman and Hall, 1998.

Syllabus

1 Ring theory (review) 1.1 General properties of rings 1.2 Characteristic of a field 1.3 Fields of fractions 1.4 Polynomials 1.5 Ideals in polynomial rings over fields

2 Factorization of polynomials (review) 2.1 Irreducibility 2.2 Tests for Irreducibility (Symmetric polynomials were considered later in the course, with the general polynomial)

3 Field Extensions 3.1 Field extensions 3.2 Simple extensions 3.3 Constructing simple extensions 3.4 Classifying simple extensions

4 The degree of an extension 4.1 The tower law (multiplicativity of extension degrees) 4.2 Algebraic Numbers

5 Ruler and Compass 5.1 Algebraic formulation 5.2 Impossibility proofs (trisection of angle of an equilateral triangle, regular septagon)

7 The idea behind Galois theory

8 Normality and separability 8.1 Splitting fields 8.2 Normality 8.3 Separability 8.4 Formal Differentiation

9 Field degrees and group orders 9.1 Linear independence of monomorphisms

10 Monomorphisms, automorphisms and normal closure 10.1 K-monomorphisms 10.2 Normal closures

11 The Galois correspondence 11.1 The fundamental theorem (proof)

12.1 A specific example (detailed examination of fundamental theorem for splitting field of $t^4 - 2$ over the rationals)

13 Soluble and simple groups 13.1 Soluble Groups (definition, and proof that S_n is solvable if and only if $n \leq 4$)

14 Solution of equations by radicals 14.2 Radical extensions; proved the theorem that polynomial equations in characteristic zero are solvable by radicals only if their Galois group is solvable 14.3 An insoluble quintic

15 The general polynomial 15.2 The general polynomial (Non-solvability by radicals of general polynomial of degree $n \geq 5$) 15.3 Solving quartic equations (Showed solvable Galois group implies an equation solvable by radicals in characteristic zero; did not discuss explicit solution of cubics and quartics using ideas from Galois theory)

16 Finite fields16.1 Structure of finite fields 16.2 Multiplicative group (also examined in detail the Galois correspondence for extensions of finite fields (left as an exercise in the text)

19 The fundamental theorem of algebra Proved the fundamental theorem of algebra using ideas from Galois theory