Math 365, Test # 2   / 100

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Instructor: Bei Hu

Name:______________________________

1. (25 points). True or False. (No proof or counter example needed).

(a). In a metric space, infinite subset of a bounded set has a limit point.

(b). In a metric space, nonempty disjoint sets are separated.

(c). In a metric space, a perfect set is uncountable.

(d). In a compact metric space, a Cauchy sequence is convergent.

(e). In a metric space, a Cauchy sequence is bounded.

2. (20 points).

(a). State PRECISELY the Heine-Borel Theorem.

(b). State PRECISELY the definition of a connected set.

(c). State PRECISELY the Monotone convergence Theorem for a sequence $\{a_n\}$.

(d). State PRECISELY the definition of the Cantor set.

3. (10 points). Let E be a connected set. Prove that its closure $\bar{E}$ is a connected set.

4. (10 points). Suppose that $0 < a_1 < 2$ and $a_{n+1} = \sqrt{2a_n}$. Prove that $\{a_n\}$ converges, and find the limit.

5. (10 points) Let X be a metric space and let $\{p_n\}$ be a Cauchy sequence. Suppose that a subsequence of $\{p_n\}$ converges in X. Prove that the sequence $\{p_n\}$ itself converges.

6. (10 points) Let X be a metric space, $\delta > 0$ and $q \in X$. Define $A = \{p|d(p,q) < \delta\}$ Prove that $\bar{A} \subset \{p|d(p,q) \leq \delta\}$. Is the equality Vail? Prove or give a counter example.

7. (5 points) Let $s_n = \sqrt{n^2 + 3n} - n$. Find

$$\lim_{n \to \infty} s_n$$

8. (10 points). Let X be a connected metric space. Suppose that E is a nonempty subset of X such that E is closed and open. Prove that $E = X$.