Math 365: Honors Analysis I Name:__________________________

Final Exam *December 11, 2000*

There are 12 problems, each worth 15 points. You will receive partial credit for all problems attempted, up to 150 points total.

1. It began to snow on a certain morning, and the snow continued to fall steadily throughout the day. At noon a snow plow started to clear a road at a constant rate in terms of the volume of snow removed per hour. The snow plow cleared 2 miles by 3 P.M. and 1 more mile by 5 P.M.. When did the snow start falling?

2. Find the general solution of $y^{(4)} - y = \cos(x)$.

3. Show that any solution of $xy'' - y' + xy = 0$ has infinitely many positive zeros.

4. Find two independent power series solutions of $y'' - xy = 0$.

5. Find two independent Frobenius series solutions of $2xy'' + y' + y = 0$.

6. a) Determine the Fourier series expansion of $f(x) = x$ on $[-\pi, \pi]$.

b) Determine the cosine series expansion of $f(x) = x$ on $[0, 1]$.

7. Derive the solution to the heat equation $a^2\dfrac{\partial^2 w}{\partial x^2} = \dfrac{\partial w}{\partial t}$ subject to the boundary conditions $w(0,t) = w_1$, $w(\pi,t) = w_2$, and $w(x,0) = f(x)$.

8. Find a function $w(r, \theta)$ that is harmonic in the unit disk, $r < 1$, and satisfies $w(1, \theta) = \theta$ for $-\pi < \theta < \pi$.

9. Solve the initial value problem $y'' - y' = \sin(x)$, $y(0) = 0$, $y'(0) = 0$, using Laplace Transforms.

10. Find the general solution of the system of equations

$$\mathbf{X}' = \begin{pmatrix} 3 & -4 \\ 2 & -3 \end{pmatrix} \mathbf{X} + \begin{pmatrix} e^t \\ e^{-t} \end{pmatrix}$$

11. Find the critical points of the system

$$\begin{aligned}\frac{dx}{dt} &= x - x^2 - xy \\ \frac{dy}{dt} &= 2y - y^2 - 3xy\end{aligned}$$

and discuss the stability of the system at each.

12. State the existence and uniqueness theorems for

   a) the solution of a general system of first order equations and

   b) the solution of a *linear* system of first order equations.