Math 365: Honors Analysis I Name:______________________________

Mid-Term Exam *October 13, 2000*

There are eight problems, each worth 15 points

1. Find the general solution of $y' + \tan(x)y = \cos(x)$.

2. Find the general solution of $y''' + y'' - 2y = x^5$.

3. Find two independent solutions of

$$y'' - x^2 y' - xy = 0$$

4. Let $f(x) = x^2$.

a) Find the Fourier series of $f(x)$ on $[-\pi, \pi]$.

b) Find the sine series of $f(x)$ on $[0, 1]$.

5. Show that any solution of Bessel's Equation

$$x^2y'' + xy' + (x^2 - p^2)y = 0$$

has infinitely positive zeros.

6. Describe the Frobenius series solutions for a second order linear differential equation and the number of such solutions that exist.

7. Let $P(x)$ and $Q(x)$ be analytic at 0 with radii of convergence $\geq R$. Prove that any power series solution of

$$y'' + P(x)y' + Q(x)y = 0$$

has a radius of convergence $\geq R$.

8. Prove that if $f(x)$ is a piecewise smooth function on $[-\pi, \pi]$, is periodic with period 2π, and is defined at points of discontinuity by

$$\frac{1}{2}[f(x^-) + f(x^+)]$$

then the Fourier series of $f(x)$ converges to $f(x)$ for all x.