Supplementary problems (assigned 8/29/03)

1. Use the ordered pair definition of functions to give a definition of 'injective' and to show that a composition of injective functions is injective.

2. Show that '+' is a well-defined operation on .

3. A very important feature of the natural numbers $\mathbf{N}$ is that every element $n = \mathbf{N}$ can be uniquely factored into primes. That is, there are unique prime numbers $p_1 < p_2 < \ldots < p_k$ and natural numbers $j_1, j_2, \ldots j_k$ such that

$$n = p_1^{j_1} p_2^{j_2} \ldots p_k^{j_k}$$

Use this fact to show that if $r \in \mathbf{Q}$ is a rational number satisfying $r^j = n$, then in fact $r \in \mathbf{Z}$.