**Supplementary problems (assigned 9/12/03)**

We define the distance between any two non-empty subsets $A$ and $B$ of a metric space $X$ by

$$(A, B) \inf\{d(x, y) : x \in A, y \in B\}.$$

(Note that the infimum exists since 0 is a lower bound for the set on the right side.)

1. Prove that $(A, B) > 0$ implies $A \cap B = \emptyset$ (not much to do here).
2. Give an example that shows we can have $(A, B) = 0$ even if $A \cap B = \emptyset$.
3. Prove that if $A$ is closed, $B$ is compact and $A \cap B = \emptyset$, then $(A, B) > 0$.