Supplementary problems (assigned 11/17/03)

1. Prove that $\cos(2x)$ is analytic at every point.

2. (Not entirely irrelevant sequence problem) Let $\{a_{i,j}\}_{i,j\in\mathbf{N}} \subset \mathbf{R}^+$ be a double sequence of positive numbers. Suppose that

   - for each fixed $i \in \mathbf{N}$ the sequence $\{a_{i,j}\}_{j\in\mathbf{N}}$ is increasing and converges to to a number $A_i \in \mathbf{R}$; and
   - $\sum_{i=0}^{\infty} A_i$ converges (call the value of the sum A).

   Show that

   - $\sum_{i=0}^{\infty} a_{i,j}$ converges for each fixed $j \in \mathbf{N}$ (call the value of the sum A_j); and
   - $\lim_{j\to\infty} A_j = A$.

   The main point here is the second item, because it involves switching two limits