Supplementary problems (assigned 9/12/03)

We define the distance between any two non-empty subsets A and B of a metric space X by

$$\operatorname{dist}(A,B) \stackrel{\text{def}}{=} \inf\{d(x,y) : x \in A, y \in B\}.$$

(Note that the infimum exists since 0 is a lower bound for the set on the right side.)

1. Prove that $\operatorname{dist}(A,B) > 0$ implies $A \cap B = \emptyset$ (not much to do here).
2. Give an example that shows we can have $\operatorname{dist}(A,B) = 0$ even if $A \cap B = \emptyset$.
3. Prove that if A is closed, B is compact and $A \cap B = \emptyset$, then $\operatorname{dist}(A,B) > 0$.