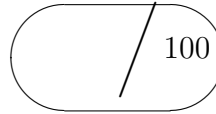


Math 366, Test # 2



April 9, 1997
Instructor: Bei Hu

Name: _____

1. (30 points).

(a). State PRECISELY the inverse function theorem.

(b). State PRECISELY the Stone-Weierstrass theorem.

(c). State PRECISELY the definition of a total derivative of a function $f : R^n \rightarrow R^m$.

(d). State PRECISELY the definition of Gamma function.

(e). State PRECISELY the definition of the Fourier series of a 2π periodic function.

(f). State PRECISELY Bessel's inequality.

2. (15 points). State and prove the contraction mapping principle.

3. (10 points). Suppose that \vec{f} is a differentiable mapping of R^1 into R^3 such that $|\vec{f}| = 1$ for every t . Prove that $\vec{f}'(t) \cdot \vec{f}(t) = 0$.

4. (10 points)

Let

$$\begin{aligned}f_1(x_1, x_2, y_1, y_2, y_3) &= 2e^{x_1} + x_2y_2 - 6y_1y_3 - 3 \\f_2(x_1, x_2, y_1, y_2, y_3) &= x_2 \cos(3x_1) + \sin x_1 + 2y_1 - y_2.\end{aligned}$$

- (1) Find the matrix for the derivative of $\vec{f} = (f_1, f_2)$ at the point $\vec{a} = (0, 1)$.
- (2) If $\vec{b} = (0, 1, 1)$, then $\vec{f}(\vec{a}, \vec{b}) = 0$. Find the partial derivatives $\vec{f}_x(\vec{a}, \vec{b})$ and $\vec{f}_y(\vec{a}, \vec{b})$.
- (3) If we want to solve $\vec{f}(x, y) = 0$ for $x = g(y)$, is the implicit function theorem valid near (\vec{a}, \vec{b}) ?

5. (10 points). Suppose that $f(x)f(y) = f(x + y)$, where f is differentiable and nonzero. Prove that

$$f(x) = e^{cx}$$

for some constant c .

6. (10 points).

Suppose that $\{f_n(x)\}$ is a uniformly bounded Riemann integrable sequence of functions on $[0, 1]$.

Let

$$F_n(x) = \int_0^x f_n(t) dt \quad 0 \leq x \leq 1.$$

Prove that there exists a subsequence $\{F_{n_k}\}$ which converges uniformly on $[0, 1]$.

7.

(a) (10 points). If f is continuous on $[0, 1]$ and

$$\int_0^1 f(x)x^n dx = 0 (n = 0, 1, 2, 3, \dots,)$$

prove that $f(x) \equiv 0$ on $[0, 1]$.

(b) (5 points). Prove the same if we only assume

$$\int_0^1 f(x)x^n dx = 0 (n = 100, 101, 102, 103 \dots).$$