Math 366, Test # 2 ___ / 100

April 9, 1997
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Name:________________________________

1. (30 points).

(a). State PRECISELY the inverse function theorem.

(b). State PRECISELY the Stone-Weierstrass theorem.

(c). State PRECISELY the definition of a total derivative of a function $f : R^n \to R^m$.

(d). State PRECISELY the definition of Gamma function.

(e). State PRECISELY the definition of the Fourier series of a 2π periodic function.

(f). State PRECISELY Bessel's inequality.

2. (15 points). State and prove the contraction mapping principle.

3. (10 points). Suppose that $\vec{f}$ is a differentiable mapping of R^1 into R^3 such that $|\vec{f}| = 1$ for every t. Prove that $\vec{f}'(t) \cdot \vec{f}(t) = 0$.

4. (10 points)

Let

$$\begin{aligned} f_1(x_1, x_2, y_1, y_2, y_3) &= 2e^{x_1} + x_2 y_2 - 6 y_1 y_3 - 3 \\ f_2(x_1, x_2, y_1, y_2, y_3) &= x_2 \cos(3x_1) + \sin x_1 + 2y_1 - y_2. \end{aligned}$$

(1) Find the matrix for the derivative of $\vec{f} = (f_1, f_2)$ at the point $\vec{a} = (0, 1)$.

(2) If $\vec{b} = (0, 1, 1)$, then $\vec{f}(\vec{a}, \vec{b}) = 0$. Find the partial derivatives $\vec{f}_x(\vec{a}, \vec{b})$ and $\vec{f}_y(\vec{a}, \vec{b})$.

(3) If we want to solve $\vec{f}(x, y) = 0$ for $x = g(y)$, is the implicit function theorem valid near $(\vec{a}, \vec{b})$?

5. (10 points). Suppose that $f(x)f(y) = f(x+y)$, where f is differentiable and nonzero. Prove that

$$f(x) = e^{cx}$$

for some constant c.

6. (10 points).

Suppose that $\{f_n(x)\}$ is a uniformly bounded Riemann integrable sequence of functions on $[0, 1]$. Let

$$F_n(x) = \int_0^x f_n(t)dt0 \leq x \leq 1.$$

Prove that there exists a subsequence $\{F_{n_k}\}$ which converges uniformly oin $[0, 1]$.

7.

(a) (10 points). If f is continuous on $[0,1]$ and

$$\int_0^1 f(x)x^n dx = 0 (n = 0, 1, 2, 3, \cdots,)$$

prove that $f(x) \equiv 0$ on $[0,1]$.

(b) (5 points). Prove the same if we only assume

$$\int_0^1 f(x)x^n dx = 0 (n = 100, 101, 102, 103 \cdots,).$$