Math 366: Honors Analysis II Name:____________________
Mid-Term Exam *March 9, 2001*

1. (20 pts) Give precise definitions for the following concepts.

a) uniformly continuous

b) convolution

c) inner product

d) metric space

e) limit point of a set in a metric space

f) open set in a metric space

g) closed set in a metric space

h) compact set in a metric space

i) complete set in a metric space

j) a continuous function $f : M \to N$ of metric spaces

2. (20 pts)
a) State the Weierstrass Approximation Theorem.

b) State the Arzela-Ascoli Theorem.

You need complete only 5 of the next 6 proofs.

3. (15 pts) Prove that if $\{f_n\} \subset C[a,b]$ and $f_n \to f$ uniformly, then $f \in C[a,b]$.

4. (15 pts) Prove that if f is analytic at 0 with radius of convergence R, then f is analytic at any point a with $|a| < R$.

5. (15 pts) Prove the Cauchy-Schwartz Inequality.

6. (15 pts) Let M be the metric space $C[a, b]$ with the sup-norm. Prove that if $A \subset M$ is compact, then A is equicontinuous.

7. (15 pts) Prove that a compact subset of a metric space must be bounded.

8. (15 pts) Prove that if $f : M \to N$ is a continuous function of metric spaces, then $\forall\, 1/m$, $\exists\, 1/n$ such that

$$d_M(x, y) < \frac{1}{n} \Rightarrow d_N(f(x), f(y)) < \frac{1}{m}$$