

**Math 366: Honors Analysis II**  
**Quiz 4** *March 30, 2001*

Name: \_\_\_\_\_

In the following problems,  $M$  and  $N$  are metric spaces.

1. Define:

a)  $A \subset M$  is connected.

b)  $A \subset M$  is arc-wise connected.

c)  $f : M \rightarrow N$  is a contractive mapping.

2. State:

a) Contractive Mapping Principle

b) Minkowski's Inequality

c) Existence and Uniqueness Theorem for Systems of Differential Equations

Do two of the following:

3. Prove that if  $f : M \rightarrow N$  is continuous and  $A \subset M$  is compact, then  $f(A) \subset N$  is compact.

4. Show that any  $m$ -th order system of differential equations is equivalent to a first order system.

5. Let  $M$  be the complete metric space of continuous functions  $x : [0, 1/2] \rightarrow [0, 1]$  with the sup-norm. For  $x \in M$  define

$$Tx(t) = \int_0^t 1 + x(s)^2 ds, \quad t \in [0, 1/2]$$

Show that  $T : M \rightarrow M$  and that  $T$  is a contractive mapping. (Bonus: find the fixed point of  $T$ .)