

**Math 366, Winter '03**  
**Homework 5**

**Profs Personal Problems:**

1. Consider the set  $C = \{(x, y, z) \in \mathbb{R}^3: x^2 - y^2 - z^2 = 0\}$ . At which points does  $C$  fail to be an embedded submanifold? What is the dimension and codimension of  $C$ ? Draw a picture of  $C$ .
2. Consider the function  $h: \mathbb{R} \rightarrow \mathbb{R}^2$  given by  $h(t) = (t^2, t^3)$  and the set  $C = h(\mathbb{R}) \subset \mathbb{R}^2$ .
  - At which points does  $C$  fail to be a submanifold of  $\mathbb{R}^2$ ?
  - Describe the tangent space to  $C$  at the point  $(1, -1)$ .
  - Draw a picture of  $C$ .

3. Consider the following set

$$M = \{(x, y, z, w) \in \mathbb{R}^4: x^2 - y^2 - zw = 1, zy + wx = 0\}$$

- At which points does  $M$  fail to be an embedded submanifold of  $\mathbb{R}^4$ ?
  - What is the dimension and codimension of  $M$ ?
  - Describe (i.e. give a basis for) the tangent space of  $M$  at the point  $(1, 1, 0, 0)$ .
  - Draw a picture of  $M$ . Full color. Stereo sound.
4. (One more time) Consider the function

$$f(x, y) = (x^3 - 2x^2y + y, y^3 - 2x^2).$$

Observe that  $f(1, 1) = (0, -1)$  and verify that  $f$  has a local inverse satisfying  $f^{-1}(0, -1) = (1, 1)$ . Starting with a guess of  $(x_0, y_0) = (1, 1)$ , compute two (increasingly better) approximations of the point  $(x, y) = f^{-1}(.1, -.8)$ .