Beals page 11, #6:

a) Let $A_n = [0,1]$ if $n \in$ is even and $A_n = \emptyset$ if n is odd. Then $\limsup A_n = [0,1]$ and $\liminf A_n = \emptyset$.

b) I show only that $\limsup A_n$ is measurable. The proof for $\liminf A_n$ is similar. For each $N \in$, let

$$B_N = \bigcup_{n \geq N} A_n$$

Then B_N and $B \cap_{N\in} B_N$ are measurable by assertion I on page 9 of Beals notes.

I claim that $B = \limsup A_n$. To see that this is so, let $x \in B$. Then $x \in B_N$ for every $N \in$. In other words, for each $N \in$, there exists $n \geq N$ such that $x \in A_n$. This can only be the case if $x \in A_n$ for infinitely many n (otherwise, we could let N be one larger than the maximum of those finitely many n for which $x \in A_n$, and it would follow that $x \notin B_N$). So $x \in \limsup A_n$.

In the other direction, suppose that $x \notin B$. Then $x \notin B_N$ for some $N \in$. Then $x \in A_n$ for at most N values of n. It follows that $x \notin \limsup A_n$. This and the preceding paragraph show that $x \in B$ if and only if $x \in \limsup A_n$, so the two sets are equal, and I conclude that $\limsup A_n$ is measurable.