

**Beals page 11, #7:** The function  $d(A, B)$  is symmetric in  $A$  and  $B$ , because  $A\Delta B = B\Delta A$ .

For transitivity, consider sets  $A, B, C \subset X$  and let  $x \in A\Delta C$  be any element. Say for instance (and with no loss of generality) that  $x \in A$  but  $x \notin C$ . Then if  $x \in B$ , it follows that  $x \in B\Delta C$ ; and if  $x \notin B$ , it follows that  $x \in A\Delta B$ . Either way,  $x \in A\Delta B \cup B\Delta C$ . This proves that

$$A\Delta C \subset A\Delta B \cup B\Delta C,$$

Consequently,

$$m(A\Delta C) \leq m(A\Delta B) + m(B\Delta C).$$

It follows that  $d$  is transitive and a semi-metric.