

**Beals, page 22 #9:** No, it's not always true. Consider for example the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  which is zero everywhere except on intervals of the form  $[n - 1/n^2, n + 1/n^2]$ ,  $n \in \mathbb{N}$ ; and on such an interval the graph of  $f$  is the pair of lines joining the points  $(n - 1/n^2, 0)$  and  $(n + 1/n^2, 0)$  to the point  $(n, 1)$ . Then  $f$  is continuous and  $f(n) = 1$  for every  $n \in \mathbb{N}$ , so  $\lim_{x \rightarrow \infty} f(x) \neq 0$  (in fact, the limit doesn't exist). However,

$$\int f = \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty.$$