Beals, page 25 #5: Set $g_n(x) = \inf_{j \geq n} f_j(x)$. Then for all $n \in$ and $x \in$,

- $\liminf f_n(x) = \lim g_n(x)$,
- $0 \leq g_n(x) \leq f_n(x)$,
- $g_{n+1}(x) \geq g_n(x)$

The second and third items allow us to apply the monotone convergence theorem to g_n and obtain

$$\int \liminf f_n = \int \lim g_n = \lim \int g_n = \liminf \int g_n \leq \liminf \int f_n.$$