

Beals, page 27 #1: By the monotone convergence theorem, we have

$$\infty > \lim \int f_n = \int \lim f_n = \int f.$$

So by the proposition on page 27, it follows that f is finite a.e.