Beals, page 34 #2: The assertion is false. To see this, consider the function which is zero except on closed intervals $[n - 1/4^n, n + 1/4^n]$, $n \geq 2$. And to define f on each of these intervals take $f(n) = 2^n$ and then make f linear on each of the remaining subintervals (i.e. the graph of f on the interval is a triangle of height 2^n and width $2/4^n$). Then on the one hand $\lim_{n\to\infty} f(n) = \lim 2^n = \infty$, so f is not bounded. But on the other hand,

$$\int f = \sum_{n=2}^{\infty} \frac{1}{2} \frac{2}{4^n} 2^n = \sum_{n=2}^{\infty} \frac{1}{2^n} = \frac{1}{2} < \infty.$$