Beals, page 34 #3: Let $\epsilon > 0$ be given. By Theorem 2 on page 33, there is a continuous, compactly supported function g such that $[1]g - f < \epsilon/4$. Note that in fact $[1]g_a - f_a = [1]g - f < \epsilon/4$ for all $a \in$.

Let $M > 0$ be chosen so that $g \equiv 0$ outside $[-M, M]$. Because g has compact support, it is uniformly continuous. So I can choose $\delta > 0$ so that $|x - y| < \delta$ implies that $|g(x) - g(y)| < \epsilon/8M$. I can, of course, assume that $\delta < M$, too. Therefore, if $|a| < \delta$, it follows that $g_a \equiv 0$ outside $[-2M, 2M]$ and

$$\begin{aligned}
[1]f - f_a &\leq [1]f - g + [1]g - g_a + [1]f_a - g_a \\
&< \frac{\epsilon}{4} + \int_{-2M}^{2M} (g(x) - g(x - a)) + \frac{\epsilon}{4} \\
&< \frac{\epsilon}{2} + \int_{-2M}^{2M} \frac{\epsilon}{8M} = \epsilon.
\end{aligned}$$

This proves that $\lim_{a \to 0} [1]f - f_a = 0$.