

*.3in Beals page 5, #4: The inequality

$$m^*(A_1 \cup A_2) \leq m^* A_1 + m^* A_2$$

holds for all $A_1, A_2 \subset \mathbb{R}$, so it remains for me to use the hypothesis to prove the reverse inequality.

Given $\epsilon > 0$, let I be a finite or countable collections of intervals covering $A_1 \cup A_2$ and satisfying

$$|I| \leq m^*(A_1 \cup A_2) + \epsilon.$$

For each interval $I \in I$, let $I' = I \cap A_1$ and $I'' = I \cap A_2$. Note that itemize

For any $I \in I$, the corresponding sets I' and I'' are open intervals satisfying

$$|I'| + |I''| \leq |I|.$$

$I' \in I$ covers A_1 .
 $I'' \in I$ covers A_2 .