Beals page 5, #7: The trick here is to remove smaller and smaller fractions of the remaining intervals as the construction progresses. Specifically, I define a decreasing sequence $\{C_k\}$ of closed sets $C_k \subset [0,1]$ as follows. First I choose a sequence $\{a_j\}_{j\in}$ of positive real numbers such that

$$s := \sum_{j=1}^{\infty} a_j < 1.$$

I let $C_0 = [0,1]$ and $D_0 = [0,1] - C_0 = \emptyset$. I divide C_0 into two closed intervals of equal length by removing an open interval of length a_1 centered at $1/2$. I call the union of the remaining closed intervals C_1 and set $D_1 = [0,1] - C_1$.

Likewise, given a closed set $C_k \subset [0,1]$ consisting of 2^k closed, pairwise disjoint intervals I of equal length, I create the set $C_{k+1} \subset C_k$ by removing from each interval $I \subset C_k$ an open interval J centered on the midpoint of I such that $|J| \le a_{k+1}|I|$. Thus, C_{k+1} consists of 2^{k+1} closed, pairwise disjoint intervals. Moreover, since the sum of the lengths of the closed intervals comprising C_k is no greater than one, it follows that the sum of the lengths of the intervals removed from C_k to create C_{k+1} is no larger than a_{k+1}. Stated in terms of the complements D_k and D_{k+1} of C_k and C_{k+1} in $[0,1]$, I have

$$m^* D_{k+1} \le m^* D_k + a_{k+1}.$$

Now if I let $C = \bigcap_{k\in} C_k$ and $D = \bigcup_{k\in} D_k$, I have $C \cup D = [0,1]$. Hence

$$m^*C \ge 1 - m^*D \ge 1 - \sum_{k=0}^{\infty} m^*(D_k - D_{k-1}) \ge 1 - \sum_{k=0}^{\infty} a_k = 1 - s > 0.$$