

Beals page 5, #9: Note that m^*A is finite because A is bounded. Suppose, in order to obtain a contradiction, that $m^*A \neq 0$. Then (by setting $\epsilon = m^*A > 0$), I can find a countable collection $\mathcal{I} = \{I_k\}_{k \in \mathbb{N}}$ of open intervals covering A such that $|\mathcal{I}| < m^*A + \epsilon = 2m^*A$. But then

$$m^*A \leq \sum_{k=1}^{\infty} m^*(A \cap I_k) \leq \frac{1}{2} \sum_{k=1}^{\infty} |I_k| = \frac{1}{2} |\mathcal{I}| < m^*A.$$

This contradiction proves that $m^*A = 0$.