Rudin, page 165/22:

Let $\epsilon > 0$ be given. By Exercise 12 from Chapter 6, there is a continuous function $h : [a, b] \to$ such that $[2]h - f < \sqrt{\epsilon/4}$. In other words,

$$\int_a^b |h(x) - f(x)|^2 \, dx < \epsilon/4.$$

By Weiestrass' Approximation Theorem, there is a polynomial P such that

$$|P(x) - h(x)| < \sqrt{\frac{\epsilon}{4(b-a)}}$$

for all $x \in [a, b]$. Thus

$$\int_a^b |P(x) - h(x)|^2 \, dx < (b-a)\frac{\epsilon}{4(b-a)} = \epsilon/4.$$

Finally,

$$|P(x) - f(x)|^2 \leq (|P(x) - h(x)| + |h(x) - f(x)|)^2 \leq 2(|P(x) - h(x)|^2 + |h(x) - f(x)|^2),$$

so

$$\int_a^b |P(x) - f(x)|^2 \, dx < 2(\epsilon/4 + \epsilon/4) = \epsilon.$$

If I now choose a sequence $\epsilon_n > 0$ tending to 0 and let P_n be the corresponding polynomials, then it follows that

$$\lim_{n\to\infty} \int_a^b |P_n(x) - f(x)|^2 \, dx = 0$$